

NAG Toolbox for MATLAB

f08jk

1 Purpose

f08jk computes the eigenvectors of a real symmetric tridiagonal matrix corresponding to specified eigenvalues, by inverse iteration.

2 Syntax

```
[z, ifailv, info] = f08jk(d, e, m, w, iblock, isplit, 'n', n)
```

3 Description

f08jk computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, by inverse iteration (see Jessup and Ipsen 1992). It is designed to be used in particular after the specified eigenvalues have been computed by f08jj with **order** = 'B', but may also be used when the eigenvalues have been computed by other functions in Chapters F08 or F02.

If T has been formed by reduction of a full real symmetric matrix A to tridiagonal form, then eigenvectors of T may be transformed to eigenvectors of A by a call to f08fg or f08gg.

f08jj determines whether the matrix T splits into block diagonal form:

$$T = \begin{pmatrix} T_1 & & & & \\ & T_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & T_p \end{pmatrix}$$

and passes details of the block structure to this function in the arrays **iblock** and **isplit**. This function can then take advantage of the block structure by performing inverse iteration on each block T_i separately, which is more efficient than using the whole matrix.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Jessup E and Ipsen I C F 1992 Improving the accuracy of inverse iteration *SIAM J. Sci. Statist. Comput.* **13** 550–572

5 Parameters

5.1 Compulsory Input Parameters

1: **d(*)** – double array

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

The diagonal elements of the tridiagonal matrix T .

2: **e(*)** – double array

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

The off-diagonal elements of the tridiagonal matrix T .

3: **m** – **int32 scalar**

m , the number of eigenvectors to be returned.

Constraint: $0 \leq m \leq n$.

4: **w(*)** – **double array**

Note: the dimension of the array **w** must be at least $\max(1, n)$.

The eigenvalues of the tridiagonal matrix T stored in **w**(1) to **w**(m), as returned by f08jj with **order** = 'B'. Eigenvalues associated with the first sub-matrix must be supplied first, in nondecreasing order; then those associated with the second sub-matrix, again in nondecreasing order; and so on.

Constraint: **w**(i) $\leq$ **w**($i + 1$) if **iblock**(i) = **iblock**($i + 1$), for $i = 1, 2, \dots, m - 1$.

5: **iblock(*)** – **int32 array**

Note: the dimension of the array **iblock** must be at least $\max(1, n)$.

The first m elements must contain the sub-matrix indices associated with the specified eigenvalues, as returned by f08jj with **order** = 'B'. If the eigenvalues were not computed by f08jj with **order** = 'B', set **iblock**(i) to 1, for $i = 1, 2, \dots, m$.

Constraint: **iblock**(i) $\leq$ **iblock**($i + 1$), for $i = 1, 2, \dots, m - 1$.

6: **isplit(*)** – **int32 array**

Note: the dimension of the array **isplit** must be at least $\max(1, n)$.

The points at which T breaks up into sub-matrices, as returned by f08jj with **order** = 'B'. If the eigenvalues were not computed by f08jj with **order** = 'B', set **isplit**(1) to **n**.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The first dimension of the array **d** and the second dimension of the array **d**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix T .

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldz, work, iwork

5.4 Output Parameters

1: **z(ldz,*)** – **double array**

The first dimension of the array **z** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, m)$

The m eigenvectors, stored as columns of Z ; the i th column corresponds to the i th specified eigenvalue, unless **info** > 0 (in which case see Section 6).

2: **ifailv(*)** – **int32 array**

Note: the dimension of the array **ifailv** must be at least $\max(1, m)$.

If **info** = $i > 0$, the first i elements of **ifailv** contain the indices of any eigenvectors which have failed to converge. The rest of the first m elements of **ifailv** are set to 0.

3: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **d**, 3: **e**, 4: **m**, 5: **w**, 6: **iblock**, 7: **isplit**, 8: **z**, 9: **ldz**, 10: **work**, 11: **iwork**, 12: **ifailv**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , then i eigenvectors (as indicated by the parameter **ifailv** above) each failed to converge in five iterations. The current iterate after five iterations is stored in the corresponding column of **z**.

7 Accuracy

Each computed eigenvector z_i is the exact eigenvector of a nearby matrix $A + E_i$, such that

$$\|E_i\| = O(\epsilon)\|A\|,$$

where ϵ is the *machine precision*. Hence the residual is small:

$$\|Az_i - \lambda_i z_i\| = O(\epsilon)\|A\|.$$

However, a set of eigenvectors computed by this function may not be orthogonal to so high a degree of accuracy as those computed by f08je.

8 Further Comments

The complex analogue of this function is f08jx.

9 Example

```
side = 'Left';
uplo = 'L';
trans = 'No transpose';
a = [2.07, 0, 0, 0;
     3.87, -0.21, 0, 0;
     4.2, 1.87, 1.15, 0;
     -1.15, 0.63, 2.06, -1.81];
vu = 0;
il = int32(1);
iu = int32(2);
abstol = 0;
[a, d, e, tau, info] = f08fe(uplo, a);
[m, nsplit, w, iblock, isplit, info] = f08jj('I', 'B', 0, 0, il, iu,
abstol, d, e);
[z, ifailv, info] = f08jk(d, e, m, w, iblock, isplit)

z =
    0.5658    -0.2328
    0.6869    -0.1626
   -0.4396    -0.3017
    0.1217     0.9101
ifailv =
         0
```

info =	0
	0
